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Implementation of multires package for determining of Sylvester type matrices

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1. Abstract:

The focus of this paper has been on multigraded polynomial system. we have illustrated the implementation of the multires package on some example of over-constrained multigraded polynomial system. The results shows that from the mixed volume, the size of the matrix that actually be derived for sparse polynomial may be generally is smaller than tha size of the matrix obtained by the greedy implementation (multires package).

2. Key words:

Multires package,spersultant function,Newton polytopes, optimal matrix, mixed volume, mixed subdivision.

3. Introduction:

The method of determine the mixed subdivision can be applied to determine the rows and columns of associated sparse resultant matrix of a polynomial system. The maple multires package was used as a tool for computing the matrix of a nonlinear polynomial system in this section. The size of the matrix is same as the number of roots for the polynomial system. All the results obtained are the sparse matrices of greedy version, because the algorithm in multires is named greedy by Pederson [2]. The greedy variant of subdivision starts with a single row, corresponding to some integer points, and proceeds iteratively by adding new rows and columns. For a given set of rows, the column set comprises all columns required to express the row polynomials. For a given set of columns, the rows are updated to correspond to the same set. The algorithm continues by adding rows and corresponding columns until a square matrix has been obtained.

Then Canny and Emiris [1] proposed a general algorithm to compute the sparse resultant of n + 1 non-homogeneous polynomials in n variables. To construct sparse resultant matrix, their algorithm used a mixed polyhedral subdivision of the Minkowski sum of the Newton polytopes [9]. This algorithm is the first general and efficient algorithm to compute the sparse resultant. Then the algorithm expanded by defining matrix entries equal to the input coefficients, whose determinant is a nontrivial multiple of the sparse resultant.

The method of constructing the sparse resultant matrices is not within the scope of this paper. However, in this work we have used the multires package [3] to implement the greedy variant algorithm on some over constrained polynomial system, in particular the multigraded system we focused on the multigraded system to investigate whether the multireas algorithm can give optimal Sylvester type matrix [10] , that is Sylvester type formula in which the determined of the resultant matrix gives exact resultants. We compare the results on the size of the matrix obtained with the mixed volume to test if the optimality conditions have been satisfied.

Example 1:

Consider the following well-constrained system (3 polynomials in 3 variables):

$f_1 = x^2 - xz^2 + yx + z - 1$, $f_2 = xyz - 3$ and $f_3 = 2x - xy - z$

```
> `toric/polytope`([x^2-x*z^2+y*x+z-1,x*y*z-3,2*x-x*y-z],[x,y]);
working on 3 polynomials in 2 variables
```

$$\left[\left[\begin{array}{cccc} 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{cc} 0 & 1 \\ 0 & 1 \end{array}\right] \left[\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 0 & 1 \end{array}\right]\right], \left[\ \{1, 3, 4\}\ \ \{1, 2\}\ \ \{1, 2, 3\}\ \right]$$

The left side is the support of each polynomial in array form, and the rirgt side is the Newton polytope or the convex hull of the support of above polynomial system.

```
> spresultant([x^2-x*z^2+y*x+z-1,x*y*z-3,2*x-x*y-z],[x,y]);
```

working on 3 polynomials in 2 variables

$$\begin{bmatrix} -3 & 0 & Z & 0 & 0 \\ 0 & -3 & 0 & 0 & Z \\ -Z & 2 & -1 & 0 & 0 \\ -1+Z & -Z^2 & 1 & 1 & 0 \\ 0 & -Z & 0 & 2 & -1 \end{bmatrix}$$

This is polynomial system of 3×3 means (3 polynomial in 3 variables) ,and the function spresultant computes the sparse matrix as shown above by using technique of hidden variable (here taking z as the hidden variable) , because in the function spresultant only consider a system of n +1 polynomials in n variables, which is called over-constrained polynomials. Here the sparse matrix is of size 5 (square matrix) ,) the computation of the mixed volume of the Newton polytopes is 5.

So the total degree of sparse resultant is equal to the mixed volume of the Newton polytopes;

$$MV_{-1} + MV_{-2} + MV_{-3} = MV(Q_1,Q_2) + MV(Q_1,Q_3) + MV(Q_2,Q_3) = 2 + 2 + 1 = 5$$

Then the computation shows that the size of sparse matrix is 5 × 5, and this means that its mixed volume expressed the sparseness of the system. It is clear that the number of solutions of the last system is 5. The BKK bound [5] serve as the lower bound on the size of matrix. From the definition of the optimal matrix we find that this system is an example of optimal matrix of Sylvester-type.

Now we will examine the generic multihomogeneous polynomial system of type (1,1;2,2) (multigraded system).

Example 2:

Consider the generic polynomial system :

$$\textit{For } i = 0, 1, 2 \qquad c_{i,00} + c_{i,01}y + c_{i,02}\, y^2 + c_{i,10}\, x + c_{i,11}\, xy + c_{i,20}\, x^2.$$

Let: for i = 0, $c_{0,00} = c_{0,10} = a_{10}$, $c_{0,01} = c_{0,11} = a_{11}$, $c_{0,02} = c_{0,20} = a_{12}$,
for i = 1 , $c_{1,00} = c_{1,10} = a_{20}$, $c_{1,01} = c_{1,11} = a_{21}$, $c_{1,02} = c_{1,20} = a_{22}$,
and for i = 2 , $c_{2,00} = c_{2,10} = a_{30}$, $c_{2,01} = c_{2,11} = a_{31}$, $c_{2,02} = c_{2,20} = a_{32}$.

```
>`toric/polytope`([a10+a11*y+a12*y^2+a10*x+a11*x*y+a12*x^2,a20+a21*y+a22*y^2+a20*x+a21*x*y+a22*x^2,a30+a31*y+a32*y^2+a30*x+a31*x*y+a32*x^2],[x,y]);
```

working on 3 polynomials in 2 variables

$$\left[\left[0\ 1\ 0\ 2\ 0\ 1 \right] \left[0\ 1\ 0\ 2\ 0\ 1 \right] \left[0\ 1\ 0\ 2\ 0\ 1 \right] \right]$$

The left side is the support of each polynomial in array form, and the right side is the Newton polytopes or the convex hull of the support of the given polynomials.

Now we compute sparse resultant matrix for the given system by using spresultant function as follow:

```
>spresultant([a10+a11*y+a12*y^2+a10*x+a11*x*y+a12*x^2,a20+a21*y+a22*y^2+a20*x
      +a21*x*y+a22*x^2,a30+a31*y+a32*y^2+a30*x+a31*x*y+a32*x^2],[x,y]);
```

working on 3 polynomials in 2 variable

$$\begin{bmatrix}
a30 & a31 & a32 & 0 & 0 & a30 & a31 & 0 & 0 & a32 & 0 & 0 & 0 & 0 \\
0 & a30 & a31 & a32 & 0 & 0 & a30 & a31 & 0 & 0 & a32 & 0 & 0 & 0 \\
a10 & a11 & a12 & 0 & 0 & a10 & a11 & 0 & 0 & a12 & 0 & 0 & 0 & 0 \\
0 & a10 & a11 & a12 & 0 & 0 & a10 & a11 & 0 & 0 & a12 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a30 & a31 & a32 & 0 & a30 & a31 & 0 & a32 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a30 & a31 & a32 & 0 & a30 & a31 & 0 & a32 & 0 \\
0 & 0 & 0 & 0 & a10 & a11 & a12 & 0 & a10 & a11 & 0 & a12 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a10 & a11 & a12 & 0 & a10 & a11 & 0 & a12 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a30 & a31 & a32 & a30 & a31 & a32 \\
a20 & a21 & a22 & 0 & 0 & a20 & a21 & 0 & 0 & a22 & 0 & 0 & 0 & 0 \\
0 & a20 & a21 & a22 & 0 & 0 & a20 & a21 & 0 & 0 & a22 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a20 & a21 & a22 & 0 & a20 & a21 & 0 & a22 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & a20 & a21 & a22 & 0 & a20 & a21 & 0 & a22 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a20 & a21 & a22 & a20 & a21 & a22
\end{bmatrix}$$

The result shows that the sparse matrix of size 14 (greedy version). However, the computations of the sum of mixed volumes of its Newton polytopes in subdivision yield 12. Here the system is unmixed and the mixed volume of any two polynomials is 4.So, the resultant matrix has at least 4 rows coming from each polynomial, sparse resultant's total degree is the sum of mixed volumes = 12.

i.e $MV(Q_1,Q_2) + MV(Q_1,Q_3) + MV(Q_2,Q_3) = 4 + 4 + 4 = 12.$

So, this result leads the size of matrix is 12×12. Therefore, it must have 12 rows to be exact resultant matrix. Hence the Sparse resultant gives matrix of minimum size (i.e. optimal matrix).

Example 3:

Consider the following polynomial system :

$f_1 = -1 + 2xy - 3x^2y - 5x$, $f_2 = 7y - 11x^2y^2 + 13x^2y - 17x$ and $f_3 = -19 - 23y - xy + 3x$.

To know the support of each polynomial in array form and the convex hull of the support of above polynomial system, we use the following function.

```
>`toric/polytope`([-1+2*x*y-3*x^2*y-5*x, 7*y-11*x^2*y^2+13*x^2*y-17*x,-19-23*y-x*y+3*x],[x,y]);
```

working on 3 polynomials in 2 variables

$$\left[\begin{bmatrix} 0 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix} \right]$$

The spresultant function gives the following matrix:

```
> sparseMatrix:=spresultant([-1+2*x*y-3*x^2*y-5*x, 7*y-11*x^2*y^2+13*x^2*y-17*x,-19-23*y-x*y+3*x],[x,y]);
```

working on 3 polynomials in 2 variables

$$\begin{bmatrix}
7 & -17 & 0 & 0 & 0 & 13 & -11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -19 & -23 & 0 & 3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -19 & -23 & 0 & 3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 7 & 0 & -17 & 0 & 0 & 0 & 13 & -11 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -19 & -23 & 0 & 3 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -19 & -23 & 0 & 3 & -1 & 0 & 0 & 0 & 0 \\
-1 & 0 & -5 & 2 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 7 & 0 & -17 & 0 & 0 & 0 & 13 & -11 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -19 & -23 & 0 & 3 & -1 & 0 \\
0 & 0 & -1 & 0 & 0 & -5 & 2 & 0 & 0 & -3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -19 & -23 & 0 & 3 & -1 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & -5 & 2 & 0 & 0 & -3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -5 & 2 & 0 & 0 & -3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 7 & 0 & -17 & 0 & 0 & 0 & 13 & -11
\end{bmatrix}$$

This system is of 3 polynomials in 2 variables. The function spresultant gives 14 in 14 matrix for greedy version. The total degree of sparse resultant is the sum of mixed volumes 4 + 3 + 4 = 11.Therefore the size of matrix is 11 in 11, so it must have 11 rows to be the exact resultant matrix. Thus, when we consider the sparseness of the given polynomials in terms of its Newton polytopes, the method can possibly yield the matrix of minimum size.

Example 4:

Consider the polynomial system:

$$f_1 = a_{11}x^2 + a_{12}x + a_{13}xy + a_{14}\ ,\ f_2 = a_{23}xy + a_{24}\ ,\ f_3 = a_{32}x + a_{33}xy + a_{34}$$

This system is multigraded of type (1,1;2,1)

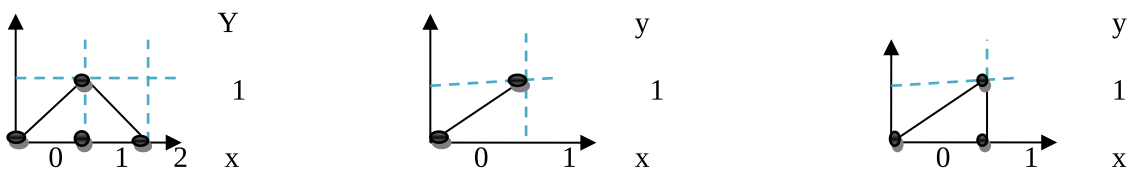

Figure (1). The Newton polytopes(convex hull) of multigraded system of type (1.1;2,1).

The support of each polynomial in array form and the convex hull of the support of this polynomial system are shown by using the following function:

>`toric/polytope`([a11*x^2+a12*x+a13*x*y+a14,a23*x*y+a24,a32*x+a33*x*y+a34],[x,y]);
working on 3 polynomials in 2 variables

$$\left[\left[\begin{bmatrix}0&1&2&1\\0&0&0&1\end{bmatrix}\begin{bmatrix}0&1\\0&1\end{bmatrix}\begin{bmatrix}0&1&1\\0&0&1\end{bmatrix}\right],\left[\{1,3,4\}\ \{1,2\}\ \{1,2,3\}\right]\right]$$

The spresultant function gives the following sparse matrix:

> spresultant([a11*x^2+a12*x+a13*x*y+a14,a23*x*y+a24,a32*x+a33*x*y+a34],[x,y]);
working on 3 polynomials in 2 variables

$$\begin{bmatrix} a24 & 0 & a23 & 0 & 0 \\ 0 & a24 & 0 & 0 & a23 \\ a34 & a32 & a33 & 0 & 0 \\ a14 & a12 & a13 & a11 & 0 \\ 0 & a34 & 0 & a32 & a33 \end{bmatrix}$$

The Maple multires package [3] gives 5×5 sparse resultant matrix. The size of this matrix equals the mixed volumes of the two-fold polynomials which given by $MV(Q_1,Q_2)$ + $MV(Q_1,Q_3)$ + $MV(Q_2,Q_3)$ = 2 + 2 + 1= 5. Since the degree of the resultant is five, the construction gives an optimal matrix. Also, if the system with this mixed supports is solvable, then by using Bernstein's theorem implies that this system has five nonzero roots.

Example 5:

Consider the following polynomials:

$f_1 = c_{11} + c_{12}\, x + c_{13} x^2 y + c_{14} xy + c_{15} x^2 + c_{16} x^3 y,$

$f_2 = c_{21} + c_{22}\, x + c_{23} x^2 y + c_{24} xy + c_{25} x^2 + c_{26} x^3 y,$

$f_3 = u_0 + u_1 x + u_2 y.$

The support of each polynomial in array form and the convex hull of the support of this polynomial system are shown by using the following function:

>`toric/polytope`([c11+c12*x+c13*x^2*y+c14*x*y+c15*x^2+c16*x^3*y,c21+c22*x+c23*x^2*y+c24*x*y+c25*x^2+c26*x^3*y,u0+u1*x+u2*y],[x,y]);

working on 3 polynomials in 2 variables

$$\left[\begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\right]$$

To compute sparse matrix we use multires package by the following spresultant function:

>spresultant([c11+c12*x+c13*x^2*y+c14*x*y+c15*x^2+c16*x^3*y,c21+c22*x+c23*x^2*y+c24*x*y+c25*x^2+c26*x^3*y,u0+u1*x+u2*y],[x,y]);

working on 3 polynomials in 2 variables

$$\begin{bmatrix}
c21 & c22 & c24 & c25 & c23 & 0 & 0 & c26 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & u0 & u2 & u1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
c11 & c12 & c14 & c15 & c13 & 0 & 0 & c16 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & u0 & u2 & 0 & u1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & u0 & u2 & 0 & u1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & c21 & 0 & c22 & c24 & 0 & c25 & c23 & 0 & 0 & c26 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & u0 & u2 & 0 & u1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & u0 & u2 & 0 & u1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & c21 & 0 & 0 & c22 & c24 & 0 & c25 & c23 & 0 & 0 & c26 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u0 & u2 & 0 & u1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u0 & u2 & 0 & u1 & 0 & 0 \\
0 & 0 & c11 & 0 & c12 & c14 & 0 & c15 & c13 & 0 & 0 & c16 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & c21 & 0 & 0 & c22 & c24 & 0 & c25 & c23 & 0 & c26 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & u0 & u2 & u1 \\
0 & 0 & 0 & 0 & c11 & 0 & 0 & c12 & c14 & 0 & c15 & c13 & 0 & 0 & c16 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & c11 & 0 & 0 & c12 & c14 & 0 & c15 & c13 & 0 & c16
\end{bmatrix}$$

The u-resultan technique is applied to form an constrained system to apply multires package. Then by applying multires package spresultant function gives matrix of size 16.

However, by mixed volume computation of its Newton polytopes the degree of the sparse resultant is 12;

$$\text{i.e,}\quad MV(Q_1,Q_2) + MV(Q_1,Q_3) + MV(Q_2,Q_3) = 4+4+4 = 12.$$

An optimal matrix of size 12 can actually be derived since the system is multigraded of type (1, 1; 3, 2). This means the optimal matix must have 12 rows to be the exact resultant matrix. If this system with this unmixed supports is solvable, it has 12 nonzero roots. Hence, sparse resultant gives matrix of minimum size (i.e. optimal matrix).

- Note that by computing the mixed volume from the mixed cells of a mixed subdivision of a polynomial system, a nonlinear system we may obtain the matrix of minimum size equals to the mixed volume.
- By using Bernstein's theorem [5] , the number of solutions of the solvable system can be predicted without explicitly solving the system.

4. TABLE (1):
SIZE OF MATRICES

Sparse Polynomial	Multires Package (Greedy Version)	Mixed Volume Computation (Sparse resultant matrix)	Remarks
Ex 1:Well constrained System (using hidden Variable technique)	5 , 6 , 6	5	Optimal matrix
Ex 2: Multigraded polynomial System of type (1,1;2,2)	14 ,15 , 21	12	

Ex 3: Over constrained Sparse resultant gives System of 3 polynomials matrix of minimum size and 2 variables	14	11
Ex 4: Multigraded polynomial System of type(1,1;2,1)	5, 6 , 7	5
Ex 5 :Multigraded polynomial System of type(1,1;3,1)	13, 14 ,16	12

5 .Notes:

We summarize the following results of the experiment in last table:

1. . From example one (not multigraded) and example four (multigrade polynomial system), we see that the computation of mixed volume of sparse resultant matrix is equal to the sparse resultant matrix' size which computed by multires package (spresultant function).This function gives three different matrices of size six, six and of size five for first example. For example four it gives four matrices from size five, six, six and seven. The smallest size of matrices of both examples is five. So, the degree of the resultant is equal to the size of smallest matrix and this mean those examples are examples of optimal matrixes of Sylvester type.

2. For the second example the computation of the mixed volume by using subdivision technique is 12 and the results of computing sparse matrix (greedy version) are three matrices of size 14, 15 and 21 .The degree of the resultant is twelve which is smaller than the size of smallest matrix that we can get by using multires package (of size 14). In this case from the mixed volume, the sparse resultant gives matrix of minimum size (i.e. optimal matrix).

3. Example three is not generic system and it is not multigraded polynomial system. The function spresultant gives just one matrix of size 14 (greedy version) and the computation of mixed volume is 11, so the degree of sparse resultant is 11.

4. Also, in example five (multigraded polynomial system of type (1,1;3,2)) the function spresultant gives matrix of size 13,14 and 16 (greedy version), so the smallest matrix is of size 13, and the computation of mixed volume is 12 . So the results for both examples show that from the mixed volume, the size of the matrix that can actually be derived for sparse polynomials is smaller than the size of the matrix obtained by (multires package).

6.CONCLUSION:

The examples two, four and five are multigraded polynomial systems and they have two groups of variables, there are at least 2! of different Sylvester type formulas .

When the given system is solvable, finding mixed volume of its corresponding Newton polytope from the volumes of the mixed cells in a subdivision, gives the number of roots for this polynomial system. We have illustrated the implementation of the multires package on some examples of over-constrained systems. The results show that from the mixed volume, the size of the matrix that can actually be derived for sparse polynomials may be generally is smaller than the size of the matrix obtained by the greedy implementation (multires package).

7.Suggestions for Further Studies:

Further work can be devoted to constructing and implementing efficient algorithms for computing support vertices, Minkowski sum and mixed volume using computer algebra system such as Maple. The conditions that can give optimal matrix needs to be further examined. Also, investigation on other types of multigraded systems in two variables with one homogenizing variable from each class and for many variables needs to be carried out.

In addition, the methods of constructing an optimal or Sylvester matrices from multigraded systems needs to be studied in the effort of finding a general algorithem that can construct a sparse resultant for every problem.

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تنفيذ حزمة متعددة الحدود لتحديد مصفوفة نوع سيلفستر

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المستخلص:

ركزت هذه الورقة على نظام كثيرات الحدود متعدد الدرجات، لقد وضحنا إجراء تطبيق التعديلات اللازمة على حزمة من الأمثلة المتركزة على نظام كثيرات الحدود متعددة الدرجات، ولقد أظهرت النتائج أنه من الحجم المختلط ، فإن حجم المصفوفة التي تم اشتقاقها فعليا لكثيرات الحدود غير الكثيفة قد يكون عموما أصغر من حجم المصفوفة التي تم الحصول عليها عن طريق التطبيق الشره (والتي سميت كذلك لأنها تحسن ما أمكن عند كل تكرار حزمة التغييرات).

الكلمات المفتاحية: حزمة التغييرات، دالة الاختزال سبارس ، متعددات السطوح نيوتن، المصفوفة المثلى (الأعظمية او القصوى)، الحجم المختلط ، التقسيم الجزئي المختلط .